Optimal experimental designs for models with a covariance function depending on the parameters of the model

Mariano Amo-Salas[1], **Jesús López-Fidalgo**[1], **and Emilio Porcu**[2]

[1] *University of Castilla-La Mancha, Spain*
[2] *University of Göttingen, Germany*

Abstract

Spatial design has become an important issue and increasing literature can be found on this subject. In particular, the presence of spatial autocorrelation has been noticed to be a serious element of complication for the design, as can be seen in a recent paper by Kiselak and Stehlik (2008). In this paper, we propose some scenarios induced by the use of covariance functions being represented by completely monotone functions on the real line (Berg and Forst, 1975). Some of them are well known in the geostatistical community as Matrn, Cauchy or Dagum models. In our approach we consider a stochastic process defined on the real line where the trend is represented through the sum of exponential components, whose parameters must be estimated. At the same time, we set the associated covariance function in order that its argument must depend on the same arguments of the trend, which is a very real situation in practice. This setting induces a typical reducibility problem whose solution is in general empty, unless one relaxes the conditions associated to the problem itself. We thus obtain a new class of positive definite functions depending on the trend parameters as well as a complete monotone function acting as generator. We illustrate the result through analytical examples and then compute optimal designs and compare them with previous results of the authors.

Keywords

Spatial design, Positive definite, Completely monotone.

References

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