Animal growth in random environments: estimation and prediction using stochastic differential equation models

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Abstract

The purpose is to study individual animal (or plant) growth in a randomly varying environment. Many deterministic models for the growth of an individual from birth to maturity size S can be written in the form of an ordinary linear differential equation $dY(t) = b(A - Y(t))dt$, where $Y(t) = h(X(t))$, with h a strictly increasing C^1 function, $A = h(S)$ and $X(t)$ the size of the individual at age t. For example, the *Gompertz* model corresponds to $h(x) = \ln x$ and the *Bertalanffy-Richards* model to $h(x) = x^c$. Solving the differential equation, one obtains some growth curve $X(t) = g(t)$ [for example, in the *Gompertz* model, $g(t) = (X(0)/S)^{\exp(-bt)}\, S$], which is usually adjusted to the observed values of $X(t)$ by nonlinear regression. This would be appropriate if the deviations between model and observations were due to measurement errors. However, this is totally inappropriate if the measuing instruments are quite precise and the deviations are mainly due to the effect of environmental fluctuations on the growth rate. In that case, the random variations should be introduced on the dynamics of the growth process. We propose the stochastic differential equation models $dY(t) = b\,(A - Y(t))\,dt + \sigma dW(t)$, where σ measures environmental noise intensity and $W(t)$ is a standard *Wiener* process. Properties of the model useful in livestock or forestry optimization are deduced, including studying the time required for an animal to reach a given size. We review the statistical issues of parameter estimation and prediction, both for one trajectory (*i.e.*, one animal) and several trajectories (*i.e.*, several animals) and show an application to cattle data. The generalization to the case where the average asymptotic size S varies from animal to animal according to a lognormal distribution will also be presented.

Keywords

Animal growth, Random environments, Stochastic differential equations, Estimation, Prediction.