# Extension of models with orthogonal block structure

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#### Abstract

An approach for the analysis and inference of unbalanced mixed models is presented. Given a mixed model  $\mathbf{Y}^0$  its extensions are the models  $\mathbf{Y} = \mathbf{L}\mathbf{Y}^0 + \mathbf{e}$  where  $\mathbf{L}$  is a matrix with linearly independent column vectors and  $\mathbf{e}$  is an error vector independent from  $\mathbf{Y}^0$ . The study is centered on core models with orthogonal block structure (van Leeeuwen et al., 1998, 1999, Fonseca et al., 2010). Crossing and nesting of core models (Fonseca et al., 2006) is carried out as well as extensions obtained using matrices  $\mathbf{L}$  such that  $\mathbf{L}^+\mathbf{L} = \mathbf{I}$ . These last extensions preserve balance in the core models.

This approach can be seen as an extension of the method for balanced random models with unequal frequencies in the last stage (Khuri and Ghosh, 1990), allowing, for instance, replicates to follow a linear correlation structure with exogenous covariates within each cell, and the use of mixed core models.

### **Keywords**

Commutative Jordan algebras, Core model, Mixed models, Orthogonal block structure, Orthogonal models.

## References

Fonseca, M., Mexia, J.T. and Zmyślony, R. (2010). Least squares and generalized least squares in models with orthogonal block structure. J. Statist. Plann. Inference 140, 71346–1352.

Fonseca, M., Mexia, J.T. and Zmyślony, R. (2006). Binary operations on Jordan Algebras and orthogonal normal models. *Linear Algebra Appl.* 417, 75–86.

Khuri, A.I. and Ghosh, M. (1990). Minimal sufficient statistics for the unbalanced two-fold nested model. *Statist. Probab. Lett.* 10, 351–353.

van Leeeuwen, D., Seely, J. and Birkes, D. (1998). Sufficient conditions for orthogonal designs in mixed linear models. J. Statist. Plann. Inference 73, 373–389.

van Leeeuwen, D., Birkes, D. and Seely, J. (1999). Balance and orthogonality in designs for mixed classification models. *Ann. Statist.* 27, 1927–1947.

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