LIBOR convexity adjustments for the Vasiček and Cox-Ingersoll-Ross models

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Abstract

In this paper we numerically implement some of the recent theoretical results concerning convexity adjustments derived within the affine term structure setup. The computation of the convexity adjustments in that setup is reduced to solving a system of ODES. Here we explore the Vasiček and Cox-Ingersoll-Ross models within LIBOR-inarrears and investigate how the convexity adjustments change with the model parameters. The two models reproduce the same behavior with the convexity adjustment showing up as an additive constant for maturity times > 5 years.

Motivation

For fixed income markets, *convexity* has emerged as an intriguing and challenging notion. Taking this effect into account correctly could provide financial institutions with a competitive advantage. The idea underlying the notion of a convexity adjustment is quite intuitive and can be easily explained in the following terms. Many fixed income products are non-standard with respect to aspects such as the timing, the currency or the rate of payment. This leads to complex pricing formulas, many of which are hard to obtain in closed-form. Examples of such products include in-arreas or in-advance products, quanto products, CMS products, or equity swaps, among others. Despite their non-standard features, these products are quite similar to plain vanilla ones whose price can either be directly obtained from the market or at least computed in closed-form. Their complexity can be understood as introducing some sort of bias into the pricing of plain vanilla instruments. That is, we may decide to use the price of plain products and adjust it somehow to account for the complexity of non-standard products. This adjustment is what is known as convexity adjustment.

Under most stochastic interest rate setups convexity adjustments cannot be computed in closed-form and market practice is to use addhoc rules or approximations when computing them. See, for instance, Hull (2006), Pugaschesky (2001), Hart (1997), Hagan (2003), Pelsser (2000), Brigo and Mercurio (2006). Most of the times one has no clue on how big this approximation error may be although there is the hope convexity adjustments would be of a different order of magnitude, when compared to market prices, making all errors negligible.

In this paper we focus on timming adjustments and, in particular, on what we define to be LIBOR in-arrears adjustments (LIA adjustments). In Gaspar and Murgoci (2008) it was shown that, in any affine term structure setting, LIBOR adjustments can be obtained in closed-form, up to the solution of a system of ODEs. Here and for the popular models of Vasiček (Vasiček, 1977) and Cox-Ingersoll-Ross (Cox, Ingersoll, and Cox, 1985) models we numerically solved the necessary systems of ODEs and show, for a reasonable range of parameter values, convexity adjustments may be substantial in terms of market quotes. This undermines some of the market practices. Trough numerical experiments we find out and discuss term structure shapes for LIA convexity adjustments.

Keywords

Affine term structure models, ODEs, Convexity.

References

Brigo, D. and Mercurio, F. (2006). *Interest Rate Models: Theory and Practice*, 2nd edition. Springer Finance, Heidelberg.

Cox, J, Ingersoll, J, and Ross, S. (1985). A theory of the term structure of interest rates. *Econometrica 53*, 385–407.

Gaspar, R.M. and Murgoci, A. (2008). Convexity adjustments for general affine term structure models. *ADVANCE working paper series, ISEG, Technical University of Lisbon.*

Hagan, P.S. (2003). Convexity conundrums: Pricing cms swaps, caps and floors. *Wilmott magazine*, 38–44.

Hart, Y. (1997). Unifying theory. *Risk*, 54–55.

Hull, J.C. (2006). *Options, Futures, and Other Derivative Securities*, 6th edition. Prentice Hall Internation, Inc.

Pelsser, A. (2000). *Efficient Methods for Valuing Interest Rate Derivatives.* Springer Finance, Heidelberg.

Pugaschevsky, D. (2001). Constant maturity products: Forward cms rate adjustments. *Risk 14(3)*, 125–128.

Vasiček, O. (1977). An equilibrium characterization of the term structure. *J. Financ. Econ. 5*, 177–188.