

On the asymptotic distribution of likelihood ratio test when parameters lie on the boundary

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Abstract

This talk discusses statistical inference dealing with the asymptotic theory of likelihood ratio tests when some parameters may lie on boundary of the parameter space. Following seminal paper by Self and Liang (1987), we derive a closed form solution for the case when one parameter of interest and one nuisance parameter lie on the boundary. The asymptotic distribution is *not* always a mixture of several chi-square distributions. For the cases when one parameter of interest and two nuisance parameters or two parameters of interest and one nuisance parameter are on the boundary, we provide an explicit solution which can be easily computed by simulation.

These results can be used in many applications, e.g. one-sided confidence intervals in environmental risk assessment and testing for random effects in genetics. Contrary to the claim of some authors in the applied literature that use of chi-square distribution with degrees of freedom as in case of interior parameters will be too conservative when some parameters are on the boundary, we show that when nuisance parameters are on the boundary, that approach may often be anti-conservative.

Keywords

Nuisance parameters on the boundary, One-sided tests, Parameters of interest on the boundary.

References

Self, S.G. and Liang, K-Y. (1987). Asymptotic properties of maximum likelihood estimators and likelihood ratio tests under nonstandard conditions. *J. Amer. Statist. Assoc.* 82, 605–610.