

About linear models: A geometric re-visitation

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Abstract

A geometric (or vector space) point of view is adopted to review some rather classic results concerning the estimation of parameters and testing the linear hypotheses in linear models. Only non-singular models are considered. An intrinsic presentation of the Best Linear Unbiased Estimators and of the Linear Hypothesis is given, this being then followed by a parameterization of the model. The relevant vector spaces are mutually related in a duality scheme. This scheme provides a profound insight into the linear nature of the model and gives an efficient way of summarizing the results. The general considerations are then applied to the Multivariate Linear Model, taking into account the tensor nature of it. The squared distance between the expectation of the observable random variables and the linear subspace specified by the linear hypothesis is clarified. It appears to be the Lawley-Hotelling trace criterion in the case of a multivariate linear hypothesis under the classic multivariate linear model.

Keywords

Gauss-Markov theorem, Loewner ordering, Lawley-Hotelling trace criterion, General linear hypothesis, Multivariate linear hypothesis.

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