Flexible sampling of semi-selfsimilar Markov processes: covariance and spectrum

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Abstract

In this paper we consider some flexible discrete sampling of a semi-selfsimilar process $\{X(t), t \in \mathbf{R}^+\}$ with scale $l > 1$. By this method we plan to have q samples at arbitrary points $\mathbf{s}_0, \mathbf{s}_1, \ldots, \mathbf{s}_{q-1}$ in interval $[1, l)$ and proceed our sampling in the intervals $[l^n, l^{n+1})$ at points $l^n\mathbf{s}_0, l^n\mathbf{s}_1, \ldots, l^n\mathbf{s}_{q-1}$, $n \in \mathbf{Z}$. Thus we have a discrete time semi self-similar process and introduce an embedded discrete time semi self similar process as $W(nq + k) = X(l^n\mathbf{s}_k)$, $q \in \mathbf{N}$, $k = 0, \ldots, q - 1$. We also consider $V(n) = \big(V^0(n), \ldots, V^{q-1}(n)\big)$ where $V^k(n) = W(nq + k)$, as an embedded q-dimensional discrete time self-similar (DT-SS) process. By introducing quasi Lamperti transformation, we find spectral representation of such process and its spectral density matrix is given. Finally by imposing wide sense Markov property for $W(\cdot)$ and $V(\cdot)$, we show that the spectral density matrix of $V(\cdot)$ and spectral density function of $W(\cdot)$ can be characterized by $\{R_j(1), R_j(0), j = 0, \ldots, q - 1\}$ where $R_j(k) = E[W(j + k)W(j)]$.

Keywords

Semi-selfsimilar process, Wide sense Markov, Multi-dimensional selfsimilar processes.

References

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