Parameter estimators for a bidimensional Ornstein-Uhlenbeck process with singular diffusion

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Abstract

The purpose of this study is to investigate the properties of an estimator of the drift parameter of a bidimensional Ornstein-Uhlenbeck process in the special case when the diffusion matrix is singular. This particular process appears in the engineering literature as a model for mechanical systems subjected to random vibrations. Usually in the literature this estimator and its properties are obtained under some regularity conditions. In particular the diffusion matrix may not be a singular matrix.

For the stochastic differential equation $dX_t = AX_t dt + B^{\frac{1}{2}} dW_t$, where $A = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{c}{m} \end{bmatrix}$, $B^{\frac{1}{2}} = \begin{bmatrix} 0 & 0 \\ \sigma & 0 \end{bmatrix}$ and W is a standard Wiener process, the maximum likelihood estimator of the drift parameter is obtained in this work. The stationarity and ergodicity of the process is studied and upon this properties, consistency and bias of the estimator are deduced.

Keywords

Ornstein-Uhlenbeck process, Stochastic differential equations, Maximum-likelihood estimator.

References

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